

# Peridynamic Modeling and Simulation of Rolling Contact Fatigue

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## Abstract

Peridynamics is used to model and simulate rolling contact fatigue problems in this paper. A damage model is implemented so that both crack initiation and crack propagation can be investigated. Although only the bond-based peridynamics is considered in this paper, the framework can be extended to the usage of state-based peridynamics.

**Keywords:** Fatigue; Simulation; Peridynamics

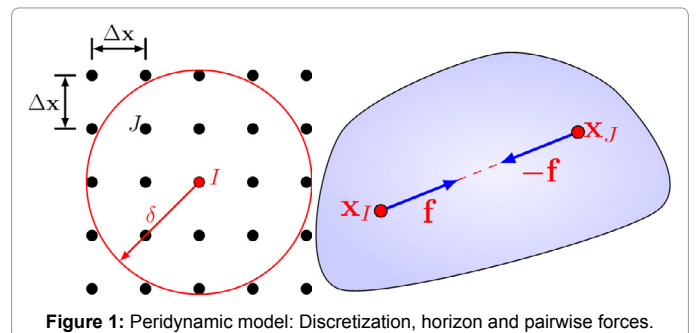
## Introduction

Bearing is one of important mechanical components in engineering designs. The rotary motion of rollers, i.e., the rolling elements, in a bearing gives rise to alternate contact between the bearing inner race and the rolling elements. High pressures are developed between the load carrying elements at the contact surface. Due to alternate contact, the contacting elements are subjected to cyclic loading and then stressing. This cyclic nature of stress makes the rolling elements and the inner race susceptible to fatigue failure. It is one of the leading causes of failure in rolling element bearings, and this phenomenon is known as Rolling Contact Fatigue (RCF) [1]. In addition to rolling element bearings, RCF is also commonly observed in gears, cam-follower mechanisms and rail-wheel contacts.

RCF manifests itself in a variety of mechanisms that ultimately leads to the final failure [2]. Subsurface originated spalling and surface originated pitting are the two most dominant mechanisms of RCF. The ultimate mode of failure depends on a number of factors including surface roughness, lubrication, load condition and others. In the mechanism of subsurface originated spalling, the micro cracks are often found to be initiated in the region of maximum shear stress below the contact surface [3]. The cracks then propagate toward the surface to form a surface spall. On the other hand, surface originated pitting occurs when surface irregularities in the form of dents or scratches are present. In this phenomenon, cracks initiate at the surface stress concentrated locations and thereafter propagate at a shallow angle to the surface [4]. When the cracks reach a critical length or depth, they branch up towards the free surface. Eventually, a piece of surface material is removed, and a pit is formed [5].

Although various models [6,7] have been successfully adopted in finite element analyses to predict rolling contact fatigue lives, researchers are always interested in new numerical methods for better and more accurate predictions. Peridynamics could be one of candidates. Peridynamics was first introduced by Silling [8] as an alternative continuum approach to solve problems involving spontaneous formations of discontinuities. The original theory was specified as the bond-based peridynamics since a pairwise force function was used to describe the interaction between a pair of material points. In the peridynamic theory, spatial derivatives were eliminated so that no special treatment on discontinuities was needed as other continuum approximations.

In the peridynamic model, the simulated domain is discretized with a number of equally spaced material points as shown in Figure 1. Each material point  $x_i$  has a horizon  $H_i$  with a radius of  $\delta$ . There are pairwise forces between the material point  $x_i$  and another material



point  $x_j$  which is in the horizon of point  $x_i$ . Consequently, the equations of motion are solved at point  $x_i$  in peridynamics as

$$\rho \ddot{u}(x_i, t) = \int_{H_i} f(\eta, \xi) dV_{x_j} + \mathbf{b}(x_i, t) \quad (1)$$

where  $\rho$  is the density,  $\ddot{u}$  is the acceleration, and  $\mathbf{b}$  is the body force.  $\eta$  and  $\xi$  are the relative displacement and the relative position respectively. They are defined as

$$\eta = x_j - x_i, x_j \in H_i \quad (2)$$

$$\xi = \mathbf{u}(x_j, t) - \mathbf{u}(x_i, t) \quad (3)$$

where  $\mathbf{u}$  is the displacement.

It shall be noted that the state-based peridynamics [9] has been developed; however, the bond-based peridynamics is employed in this paper. Therefore, it is assumed a "virtual" bond between material points  $x_i$  and  $x_j$ . The pairwise force  $f$  is calculated based on the bond strain  $S$ .

$$f(\eta, \xi) = C \times S(t, \eta, \xi) \quad (4)$$

$$S(t, \eta, \xi) = \frac{\|\eta + \xi\| - \|\xi\|}{\|\xi\|} \quad (5)$$

where  $C$  is the micromodulus.

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In the three-dimensional elasticity model, a peridynamic material is considered to be microelastic, and the micromodulus [10] is derived as:

$$C = 18k / \pi \delta^4 \quad (6)$$

where  $k$  is the material bulk modulus. In the derivation, the Poisson's ratio  $\nu$  is fixed as  $1/4$ .

Since internal forces in the governing equations (1) are calculated via integrations instead of derivatives, peridynamics has advantages to be used for solving the problems with discontinuities. It has been applied to structural stability and failure analyses [11,12], fracture mechanics [13-16], and composite materials [17-20]. In this paper, we employed peridynamics to study rolling contact fatigue. The peridynamic damage model [21] includes a damage variable, named the "remaining life" which evolves over time in each bond, to determine crack initiation and propagation.

To implement the peridynamic damage model in rolling contact fatigue life prediction, a peridynamic solid is considered to be subject to cyclic loading between two extremes, denoted  $max$  and  $min$ . For a studied bond  $\xi$  in the family of  $x_i$  in the body, the cyclic bond strain can be calculated as:

$$\varepsilon = |S^{max} - S^{min}| \quad (7)$$

Where  $S^{max}$  and  $S^{min}$  are the bond strains at the two cyclic loading extremes.

A variable,  $\lambda(x, N)$ , of "remaining life" is introduced for each bond  $\xi$  connected to any point  $x$ . The remaining life develops gradually as the loading cycle  $N$  increases based on the following relation:

$$d\lambda(N)/dN = -A\varepsilon^m \lambda(0) = 1 \quad (8)$$

where  $\varepsilon$  is the current cyclic strain in the bond as defined in equation (7),  $A$  is a positive parameter and  $m$  is a positive constant exponent. The bond breaks irreversibly at the earliest loading cycle  $N$  if the following criterion is satisfied:

$$\lambda(N) \leq 0 \quad (9)$$

It shall be noted that  $A$  and  $m$  can be determined via the strain-life curve [21].

## Discussion

In this paper, peridynamics and the fatigue model described above are adopted to study rolling contact fatigue. The simulated model, shown in Figure 2, is subject to the Hertzian pressure. The bottom of the model is fixed, and the periodic boundary conditions are applied on two sides of the model. The peridynamic material has the Young's modulus of 70 GPa and the density of 2700 kg/m<sup>3</sup>.

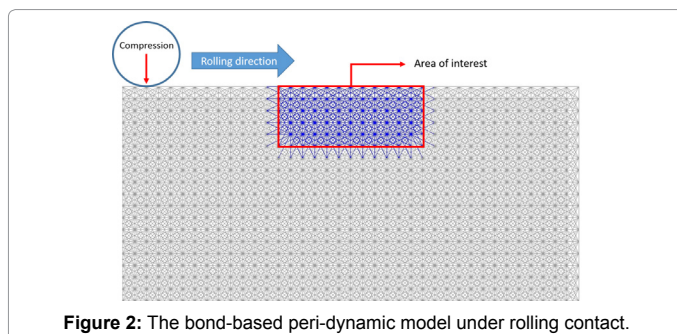


Figure 2: The bond-based peri-dynamic model under rolling contact.

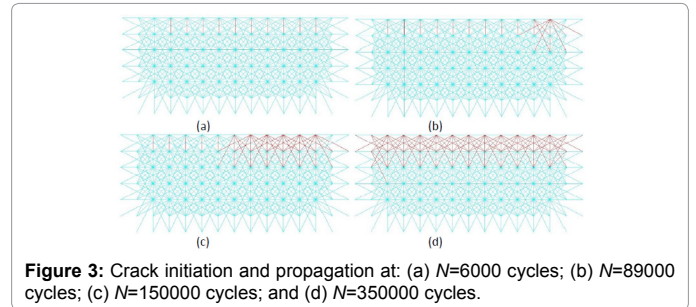


Figure 3: Crack initiation and propagation at: (a)  $N=6000$  cycles; (b)  $N=89000$  cycles; (c)  $N=150000$  cycles; and (d)  $N=350000$  cycles.

The crack initiation and propagation are shown in Figure 3 based on peridynamics simulation results. Figure 3a indicates that the crack initiation occurs below the surface at 6000 load cycles. Once a bond with the lowest fatigue life was identified, its mechanical properties were decreased to approximate a broken bond. Therefore, the fatigue crack propagation was simulated as depicted in Figures 3b-3d. It is concluded that crack started at the location with more broke bonds and then propagated to the other side of the model.

## Conclusion

In this paper, peridynamics is employed to study RCF. The proposed approach offers several advantages over existing research models for RCF. First of all, it can easily to predict fatigue crack initiation and propagation without mesh refinement. Unlike some other fracture mechanics approaches where one or more initial crack geometries and locations are assumed, the present approach does not require the existence of initial cracks or defects in the simulation domain. In fact, cracks initiation occur naturally under the contact loading and then propagate whenever the stress conditions permit. Secondly, non-homogeneous description of the material can be modeled using this approach. Therefore, studying RCF of bearing made of composites will become easy with the proposed approach. At last, the model has the potential to offer more insights into the physical mechanism of the failure process occurring during RCF.

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